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CREATION OF A SCALAR POTENTIAL IN 2D DILATON GRAVITY*

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ABSTRACT

We investigate quantum corrections of the 2-d dilaton gravity near the singularity. Our motivation comes from a s-wave reduced cosmological solution which is classically singular in the scalar fields (dilaton and moduli). As result we find, that the singularity disappears and a dilaton/moduli potential is created.

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In two dimensions (2-D) the dilaton gravity could be formulated as a quantum theory in the last years. This opens the possibility to quantize higher dimensional theories near regions where a 2-D part factorizes. As a first step one can quantize only the 2-D part and leave the dynamical fields living in the other dimensions as a classical background. An example is the s-wave reduction of higher dimensional theories with a spherical symmetry, e.g. coming from black holes or cosmological solutions. In this talk we are not going to describe the black hole background. Instead, our interest in this investigation comes from a special cosmological solution[2,3], that can be obtained by a dimensional reduction. This solution has singularities in the scalar fields (dilaton and modulus) and our aim is to discuss quantum corrections near the singularity (details can be found in Ref. 4). A crucial property of this solution is that near the singularity it factorizes in a smooth 3-D spherical part and a divergent 2-D part. If we perform a s-wave reduction we find that the divergency is controlled by the known 2-D dilaton gravity

$$S^{(2)} = \int d^2z\sqrt{g}e^{-2\phi}\left(R^{(2)} + 4(\partial\phi)^2 + \lambda\right) \tag{1}$$

where ϕ is the 2-D dilaton and λ is constant. The classical solution in conformal coordinates is given by

$$ds^2 = e^{2\sigma}dz^+dz^- \qquad , \qquad e^{-2\phi} \sim e^{-2\sigma} = u - \lambda z^+ z^- \qquad (u = const.) \tag{2}$$

The singularity of this solution is in the strong coupling region ($\phi \to +\infty$) whereas in weak coupling region ($\phi \to -\infty$) it behaves smooth.

Before we turn to the discussion of a scalar potential let us shortly summarize the quantization procedure (we are now following here the notation of de Alwis[1]). Choosing the conformal gauge: $g_{ab} = e^{2\sigma}\hat{g}_{ab}$ and performing the field redefinition

$$x = \frac{1}{\sqrt{4\kappa}}\left(-\sqrt{\kappa^2 + 4e^{-4\phi}} + \sqrt{\kappa}\text{arcsinh}\frac{\kappa}{2}e^{2\phi}\right) \ , \ y = \sqrt{\kappa}\left(\sigma - \frac{1}{\kappa}e^{-2\phi} - \phi\right) \tag{3}$$

we can write the 2d model as (including terms from the functional integration measure)

$$S^{(2)} = \int d^2z\sqrt{\hat{g}}\left[(\partial x)^2 - (\partial y)^2 + \hat{R}\,\Phi(x,y) + T(x,y)\right] \ . \tag{4}$$

However, the function $\Phi(x,y)$ and $T(x,y)$ are not arbitrary. The requirement of independence of the reference metric $\hat{g}_{ab}$ has the consequence that the 2-D theory has to be conformally invariant. The simplest choice is to take a linear dilaton Φ and a exponential tachyon T

$$\Phi = \sqrt{\kappa}y \qquad , \qquad T = \lambda\, e^{\frac{2}{\sqrt{\kappa}}(x-y)} \ . \tag{5}$$

With this choice we have a well defined 2-D quantum theory (mathematically the same as the non-critical string theory in one dimension). Now, one defines the quantum theory in terms of these x and y variables and regards Eq. (1) as the

classical limit. As solution of the equation of motion in x and y one finds (if we restrict ourselves on solutions depending on the product z^+z^- only)

$$x = y = \frac{1}{\sqrt{\kappa}}\left(u - \lambda z^+ z^-\right) \tag{6}$$

($u = const.$). Using the transformation (3) we can express this solution by ϕ and σ. In the weak coupling limit ($e^{2\phi} \ll 1$) we have the desired classical solution (2). But our original singularity appeared in the strong coupling region. In this limit ($e^{2\phi} \gg 1$) we obtain $\phi = -\frac{1}{\kappa}(u - \lambda z^+ z^-)$, $\sigma = \frac{1}{\kappa} e^{-2\phi}$. Thus, after incorporation of quantum corrections ($\sim \mathcal{O}(e^{2\phi})$) the solution becomes smooth also in the strong coupling region[1].

One can now ask, whether quantum corrections can form a potential in the scalar fields x and y (or ϕ and σ resp.). A potential in our original action (1) corresponds to an additional tachyon contribution. The tachyon we have discussed so far is only *one* possibility. The most general tachyon field is a combination of the solutions of the Weyl invariance condition, that are given by

$$\begin{aligned} &\Phi(x,y) = ax + by && \text{with} \quad a^2 - b^2 = -\kappa \quad , \\ &T(x,y) \sim e^{\alpha x + \beta y} && \text{with} \quad \tfrac{1}{2}(\alpha^2 - \beta^2) - a\alpha + b\beta - 2 = 0 \; . \end{aligned} \tag{7}$$

In order to get the right classical limit we set furthermore $a = 0$. But there is also another parameterization for the tachyon[5]. Using the mass shell condition we can replace α or β and then we can expand the tachyon field in powers of the remaining α or β. After this procedure we find an infinite set of tachyon fields which are parameterized by two integers m and n. Because the corresponding tachyon equation is linear every term of this expansion fulfills the equation, too. If we restrict ourselves on $\kappa = \frac{24-N}{6} = 4$ (i.e. $N = 0$) these additional terms are

$$T_2^{(n)} = (y - x)^n \, e^{2x} \quad , \quad T_3^{(m)} = (x \pm y)^m \, e^{2y} \; . \tag{8}$$

Instead of Eq. (7) we have now as general tachyon field $T(x,y)$

$$T(x,y) = \lambda e^{\frac{2}{\sqrt{\kappa}}(x-y)} + \sum_{(n,m)} (\mu_2^n T_2^{(n)} + \mu_3^m T_3^{(m)}) \tag{9}$$

where the function x and y are given by the Eq. (3) (the term $T_2^{(0)}$ was already discussed in Refs. 1 and 6). A remarkable property of these terms is, that they have in the classical limit ($\phi \to -\infty$) the typical non-perturbative structure: $T_{2,3} \sim e^{-\frac{1}{2}e^{-2\phi}} \sim e^{-\frac{1}{(2g_s^2)}}$, where $g_s = e^{\phi}$ is the string coupling constant. On the other side, in the strong coupling region ($\phi \to \infty$) we have: $T_2 \sim e^{4\phi} \to \infty$, $T_3 \sim e^{-2\phi} \to 0$. Therefore, these terms vanish very rapidly in the weak coupling (classical) region

and become important in the strong coupling region. Furthermore, since x and y are functions of the scalar fields these terms represent a potential in ϕ and σ. So, the quantized theory (4) differs from the classical theory (1) not only by a modification of the kinetic terms of ϕ and σ but also by an additional potential in the scalar fields. What does this mean for a cosmological solution? There, the scalar fields ϕ and σ correspond to a dilaton field and a modulus field. As we have pointed out the cosmological solution is classically singular in the scalar fields. This semi-classical quantization (we have only quantized the scalar fields) showed that the singularity disappears and, in addition, a dilaton/moduli potential is created. It remains an open question whether this potential can yield sufficient inflation. In order to discuss this question one has to transform the theory back to the 4-D Einstein frame and has to show that the resulting potential has a flat direction which in turn give an extended inflation[7]. Probably, this is possible for a suitable choice of the constants $\mu_{2,3}^{m,n}$. However, normally in discussing of non-perturbative corrections one imposes further string symmetries to restrict the possible contributions and it deserves further investigations to show that this will not destroy a flat direction.

1. References

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